18. Find the complete and Singular Solution of $z = xp + yq + p^2 - q^2$.

19. Evaluate $\iint_S \vec{F}, \vec{n}\, ds$ where $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ and S is the part of the surface of the sphere $x^2 + y^2 + z^2 = 1$ which lies in the first octant.

20. Obtain the Fourier series expansion of the function $f(x)$, defined by $f(x) = x$, $-\pi < x \leq \pi$.

APRIL/MAY 2024

CAEL22/FAEL22 — BASIC MATHEMATICS – II

Time : Three hours Maximum : 75 marks

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SECTION A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. State Leibnitz's theorem.

2. Find the n^{th} derivative of a^x.

3. Solve $(D^2 - 3D + 5)y = 0$.

4. What do you mean by complementary solution?

5. Find the Partial Differential Equation of all planes having equal intercepts from the x and y axis.

6. Define complete integral.

7. State Gauss divergence theorem.

8. Define a simple closed curve.

9. Evaluate $\int_0^{\pi/2} \cos^2 x \, dx$.

10. What is the value of the Fourier Constant a_n when odd function $f(x)$ is expanded in $(-\pi, \pi)$.

SECTION B — (5 × 5 = 25 marks)

Answer ALL the questions.

11. (a) If $x = a(\cos t + t \sin t)$ $y = a(\sin t - t \cos t)$ find $\frac{d^2y}{dx^2}$.

Or

(b) Find the maximum value of $f(x, y) = x^4 + y^4 - 4xy + 1$.

12. (a) Solve $(D^2 - 4D - 12)y = \sin x \sin 2x$.

Or

(b) Solve $(D^2 + 4)y = x^2$.

13. (a) Form the partial differential equation by eliminating a and b from $z = ax + by + a^2 + b^2$.

Or

(b) Solve $z = p^2 + q^2$ for complete and singular solutions.

14. (a) Find the value of 'a' such that $\vec{F} = (axy - z^2)\vec{i} + (x^2 + 2yz)\vec{j} + (y^2 - axz)\vec{k}$ is irrotational.

Or

(b) If $\vec{F} = 3xy\,\vec{i} - y^2\,\vec{j}$, evaluate $\int_C \vec{F}.d\vec{r}$ where C is the curve on the xy plane $y = 2x^2$ from (0, 0) to (1, 2).

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15. (a) Evaluate $\int e^{2x} \cos^3 x \, dx$.

Or

(b) Using Bernoulli's formula find $\int (2x^2 - 4x + 5)\cos 3x \, dx$.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. If $y = \sin(m \sin^{-1} x)$.

Prove that $(1 - x^2)y_2 - xy_1 + m^2 y = 0$ and $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$.

17. Solve $(D^2 - 2D + 1)y = x^2 + 1 + \sin 2x$.